$$y^{\text{iv}} = \frac{(ab_1 - a_1b)[(a + bY')^2Y^{\text{iv}} - 10b(a + bY')Y''Y''' + 15b^2(Y'')^3]}{(a + bY')^7},$$

$$y^{\text{v}} = \frac{(ab_1 - a_1b)[(a + bY')^3Y^{\text{v}} - 15b(a + bY')^2Y''Y^{\text{iv}} - 10b(a+bY')^2(Y''')^2 + [105(a + bY')b^2(Y'')^2Y''' - 105b^3(Y'')^3]}{(a + bY')^9}$$

From (2) we find

$$y' = Y - XY', \quad y'' = X^3Y'', \quad y''' = -X^4(3Y'' + XY'''),$$

$$y^{\text{iv}} = X^5[12Y'' + 8XY''' + X^2Y^{\text{iv}}]$$

$$y^{\text{v}} = -X^6[60Y'' + 60XY''' + 15X^2Y^{\text{iv}} + X^3Y^{\text{v}}].$$

It can be easily shown by substitution that each transformation leaves the given equation invariant.

Also solved by J. W. Clawson and Geo. W. Hartwell.

349. Proposed by C. N. SCHMALL, New York City.

If $y = a \cos(\log x) + b \sin(\log x)$, eliminate the constants a and b and obtain the equation

$$\frac{x^2d^2y}{dx^2} + \frac{xdy}{dx} + y = 0.$$

Solution by C. C. Steck, New Hampshire State College.

By differentiating the equation $y = a \cos(\log x) + b \sin(\log x)$, we obtain

$$\frac{xdy}{dx} = -a \sin(\log x) + b \cos(\log x).$$

Differentiating this and multiplying the resulting equation by x, we get

$$\frac{x^2d^2y}{dx^2} + \frac{xdy}{dx} = -a \cos(\log x) - b \sin(\log x) = -y.$$

From the last two equations readily follows the desired result

$$\frac{x^2d^2y}{dx^2} + \frac{xdy}{dx} + y = 0.$$

Also solved by M. E. Graber, J. B. Smith, J. W. Clawson, P. Peñalver, C. Hornung, Elmer Schuyler, A. M. Harding, W. W. Beman, A. L. McCarty, H. L. Slobin, Clifford N. Mills, Francis Rust, I. A. Barnett, F. C. Reisler, G. W. Hartwell, S. W. Reaves, Richard Morris, Albert R. Nauer, Barnem Libby, and Walter C. Eells.

351. Proposed by C. N. SCHMALL, New York City.

In the ellipse $(x^2/a^2) + (y^2/b^2) = 1$ are given e the eccentricity, and the angle φ which the normal at any point P (on the curve) makes with the major axis. If R is the radius of curvature at P prove that

$$R = \frac{a(1 - e^2)}{(1 - e^2 \sin^2 \varphi)^{\frac{3}{2}}}.$$

SOLUTION BY GEO. W. HARTWELL, Hamline University, St. Paul, Minn.

For the given ellipse

$$\frac{dy}{dx} = -\frac{b^2x}{a^2y}, \tag{1}$$

$$\frac{d^2y}{dx^2} = -\frac{b^4}{a^3y}. \tag{2}$$

Also

$$\frac{dy}{dx} = \cot \varphi. \tag{3}$$

Making these substitutions in the usual formula for radius of curvature,

$$R = \frac{(1 + \cot^2 \varphi)^{3/2}}{\frac{b^4}{a^2y^3}} = \frac{a^2y^3}{b^4 \sin^3 \varphi}. \tag{4}$$

From (1) and (3)

$$b^2x = -a^2y \cot \varphi. \tag{5}$$

Substituting this value of x in the equation of the ellipse, we have

$$a^4y^2 \cot^2 \varphi + b^2a^2y^2 = a^2b^4.$$

Hence,

$$y^2 = \frac{b^4}{b^2 + a^2 \cot^2 \varphi} = \frac{b^4 \sin^2 \varphi}{a^2 \cos^2 \varphi + b^2 \sin^2 \varphi} = \frac{b^4 \sin^2 \varphi}{a^2 - (a^2 - b^2)\sin^2 \varphi}. \tag{6}$$

But $a^2 - b^2 = a^2e^2$. Hence

$$y^3 = \frac{b^6 \sin^3 \varphi}{a^3(1 - e^2 \sin^2 \varphi)^{3/2}}$$

and

$$R = \frac{b^2}{a(1 - e^2 \sin^2 \varphi)^{3/2}}.$$

But $b^2 = a^2(1 - e^2)$. Hence,

$$R = \frac{a(1 - e^2)}{(1 - e^2 \sin^2 \varphi)^{3/2}}.$$

Also solved by RICHARD MORRIS, C. C. STECK, F. M. MORGAN, J. W. CLAWSON, S. W. REAVES, and J. G. GONZALES.

MECHANICS.

A solution of 273 was received from J. W. CLAWSON too late for the June issue.

271. Proposed by B. F. FINKEL, Springfield, Mo.

A hollow spherical shell is filled with a frictionless fluid and rolls down a rough inclined plane. After rolling t seconds the fluid suddenly solidifies. Determine the subsequent motion of the spherical shell.

SOLUTION BY J. W. CLAWSON, Collegeville, Pa.

Let m be the mass of contained fluid; m' the mass of the shell alone; α the angle of inclination of the plane; a the radius of sphere; x the distance the sphere